**JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES**
**www.journalimcms.org**

**ISSN (Online) : 2454 -7190 Vol.-19, No.-9, September (2024) pp 18 - 23 ISSN (Print) 0973-8975**

# ALGORITHM FOR FINDING DOMINATION RESOLVING NUMBER OF A GRAPH

**Iqbal M. Batiha[1], Nidal Anakira[2], Basma Mohamed[3]**

[1]Department of Mathematics, Al Zaytoonah University, Amman 11733, Jordan.

[1]Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman, UAE.

[2]Faculty of Education and Arts, Sohar University, Sohar 3111, Oman.

[2]Jadara Research Center, Jadara University, Irbid 21110, Jordan.

[3]Giza Higher Institute for Managerial Sciences, Tomah, Egypt.

Email: [1]i.batiha@zuj.edu.jo, [2]nanakira@su.edu.om, [3]bosbos25jan@yahoo.com

Corresponding Author: **Iqbal M. Batiha**

https://doi.org/10.26782/jmcms.2024.09.00003

(Received: June 19, 2024; Revised: August 25, 2024; Accepted: August 31, 2024)

## Abstract

*A minimum resolving set is a resolving set with the lowest cardinality and its cardinality is a dimension of connected graph $G = (V, E)$, represented by $dim(G)$. A dominating set $D$ is a set of vertices such that each $v$ of $G$ is either in $D$ or has at least one neighbor in $D$. The dominance number of $G$ is the lowest cardinality of such a set. The lowest cardinality of the dominant resolving set is called a dominant metric dimension of $G$, represented by $Ddim(G)$. This paper presents an algorithm for finding the domination resolving number of a graph.*

**Keywords:** Domination Number, Metric Dimension, Resolving Dominating Set.

## I. Introduction

All of the graphs studied are simple, connected, undirected, and have a large number of edges but no loops. Because of its diverse and various applications in many fields like algorithmic designs, communications networks, social sciences, and many others, the investigation of dominance is regarded as the quickest-growing subject in the theory of graphs. Ayhan A. Khalil demonstrated in [I] the domination number for the web and helm graphs, whereas C.S. Nagabhushana et al. investigated in [XI] the domination number for the windmill and friendship graphs. The dominance number for the tadpole graph was proven by K. B. Murthy [XXI]. The dominance number for the book graph and stacked book graph was proved by B.N. Kavitha and Indrani Kelkar [X]. The domination number and the dominating set of graphs, like firecracker, coconut tree, banana tree, diamond snake, and fan graphs, were discussed by A. Sugumaran and E. Jayachandran in [II]. By using the term resolving domination number, R. C. Brigham et al. [XXV] merged the concepts of metric dimension and dominating set. Susilowati et al. [XXII] studied the dominant metric dimension of a specific graph class and the dominant metric dimension of graph joint and comb products. The resolving independent domination number of fan, helm, friendship, cycle, and path graphs were presented by Mazidah et al. in [XXXVIII]. R. Alfarisi et al. [XXIV] studied and established sharp constraints of the resolving domination number of $G$, as well as the exact value of various family graphs. Kurniawati et al. [XXVI] determined the resolving domination number of friendship graphs and its operation. F. Muhamad et al. [XII] proposed a computer program for determining the basis and dimension of a graph. For more results, see [III,IV,V,VI,VII,VIII,IX].

This paper is organized as follows: In Section 2, we introduce the basic concepts. In Section 3, some new propositions are explained. In Section 4, we present an algorithm for finding the dominant basis of a graph. Lastly, the conclusion of this work is depicted in Section 5.

## II. Preliminaries

In this section, we shall recall the most important elementary definitions and basic facts needed in our study later on.

**Definition 1.** [XXVII] An open diagonal ladder $O(DL_n)$ is obtained from a diagonal ladder graph by deleting the edges $u_i v_i$, for $i = 1,2,3,\dots,n$.

**Definition 2.** [XXVII] Suppose that $G$ is a connected graph for which the ordered set $\overline{W} \subseteq V(G)$. The definition of the dominant metric dimension of $G$, which is denoted by $Ddim(G)$, is given by $Ddim(G) = min\{|\overline{W}|: \overline{W}$ is the dominant resolving set of $G\}$.

**Lemma 1.** [XIV,XXII] Let $G$ be a connected graph and $S \subseteq V(G)$. If $S$ contains a resolving set of $G$, then $S$ is a resolving set of $G$.

**Proposition 1.** [XXII]For a variety of well-known graph types, this proposition shows some results obtained from $\gamma(G)$:

1. For path $P_n$ and cycle $C_n$, $(P_n) = (C_n) = \lceil n/3 \rceil$, $(P_n) = 1$ and $(C_n) = 2$.
2. For a complete graph $K_n$, $(K_n) = 1$ and $(K_n) = n - 1$.
3. For star $S_n$, $(S_n) = 1$ and $(S_n) = n - 2$, for all $n \geq 2$.
4. For complete bipartite graph $K_m$, $\gamma(K_m, n) = 2$ and
$$dim(K_m, n) = m + n - 2,$$
for every $m, n \geq 2$.

## III. New Propositions

In what follows, we present some new propositions that will be needed to establish the desired algorithm for determining the dominant basis of graphs:

1. If $G$ are $(2, n)$ $C_4$-snake and $4\Delta_n$-snake graph, then
$\gamma_r(G) = \frac{4n-4}{5},\ n \geq 1.$
2. If $G$ is $Z - (P_n)$ graph, then $\gamma_r(G) = \frac{n}{2} - 1$.

3. If $G$ is globe graph $Gl_n$, then $\gamma_r(G) = n - 2$.
4. For chain silicate graph $CS_n$, $\gamma_r(CS_n)=\frac{n+5}{3}$, for $n \geq 4$.
5. For bistar graph $B_{3,n}$, $\gamma_r(B_{3,n}) = n - 2$, for $n \geq 2$.

## IV. Algorithm to Determine Dominant Basis of Graph

The suggested algorithm consists of three steps: the first measures the distance between two vertices, the second establishes the graph's basis, and the third is involved in verifying the dominant basis of the under-consideration graph.

In the first step of the algorithm is to compute the distance between two vertices $i$ and $j$ in graph $G$ of order $n$, choose a vertex $i$ as a beginning vertex, and a vertex $j$ as a destination vertex. Using $W = \{\{y_1\}|y_1 \in V(G)\}$ to describe the collection of all singleton subsets of $G$, examine each vertex's representation concerning $\{y_1\}$. If no two vertices in $V(G)$ have the same representation concerning $\{y_1\}$, the procedure is finished. If there are two vertices in $V(G)$ have the same representation about $\{y_1\}$, then build the new set $W = \{\{y_1, y_2\}|y_1, y_2 \in V(G)\}$, the process will be continued until we get $W = \{\{y_1, y_2, \cdots, y_j\}|y_i \in V(G), 1 \leq i \leq j$, while there are no two vertices in $V(G)$ have the same representation with regard to $\{y_1, y_2, \cdots, y_j\}$ and check whether it is a dominating set. So we can declare $\{y_1, y_2, \cdots, y_j\}$ as the dominant basis of the graph with $\left|\{y_1, y_2, \cdots, y_j\}\right|$ as the value of the dominant metric dimension graph.

---

**Algorithm (Generating Dominant Basis of a Graph)**

---

**Input:** Adjacency matrix $A[i, j]$, Distance matrix.
**Output**: Checking minimum dominant basis of graph.

Set $n$ are the order of the graph
**For** $X = 1$ to $n$ **do**
Set $\beta$ as a collection of all subsets of $G$ with $|X| = q, X \in \beta$
**For** $X$ in $\beta$ **do**
If there are no two vertices have the same representation concerning $X$
Set $X$ as the basis and $q$ as the metric dimension
Count= $X$
**For** $i = 1$ to $n$ **do**
**If** $X[i] = 1$
**For** $j = i + 1$ to $n$ **do**
**If** $X[j] = 0$
$\text{Count}[j] = \text{Count}[j] + A[i, j]$
**End if**
**End for**
Flag $=\prod(count > 0)$
**If** Flag $= 1$
Print-dominated basis of graph
Exit.

## V. Conclusion

Metric dimension has several applications in robot navigation, network discovery, and verification, application to wireless sensor network localization, image processing, and combinatorial optimization. In this paper, we presented an algorithm for finding the dominant basis of the graph, which can be merged with many recent contributions found in references [XV,XVI,XVIII,XIX] and applied to many applications such as applications discussed in references [XIII,XVII,XX].

## VI. Acknowledgements

We appreciate the time and effort that the reviewers dedicated to providing feedback on our manuscript and are grateful for the insightful comments on and valuable improvements to our paper.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References

I. A. A. Khalil. : 'Determination and testing the domination numbers of Helm graph, web graph and Levi graph using MATLAB'. *Journal of Education Science*. Vol. 24, pp. 103-116, 2011. https://www.iasj.net/iasj/download/2b430f4e0c4f89fd

II. A. Sugumaran, E. Jayachandran. : 'Domination number of some graphs'. *International Journal of Scientific Development and Research*. Vol. 3, pp. 386-391, 2018. https://api.semanticscholar.org/CorpusID:213194763

III. B. Mohamed. : 'A comprehensive survey on the metric dimension problem of graphs and its types'. *International Journal of Theoretical and Applied Mathematics*. Vol. 9, pp. 1-5, 2023. 10.11648/j.ijtam.20230901.11

IV. B. Mohamed, L. Mohaisen, M. Amin. : 'Binary equilibrium optimization algorithm for computing connected domination metric dimension problem'. *Scientific Programming*. Vol. 2022, pp. 1-15, 2022. 10.1155/2022/6076369

V. B. Mohamed, L. Mohaisen, M. Amin. : 'Computing connected resolvability of graphs using binary enhanced Harris Hawks optimization'. *Intelligent Automation & Soft Computing*. Vol. 36, pp. 2349-2361, 2023. 10.32604/iasc.2023.032930

VI. B. Mohamed, M. Amin. : 'A hybrid optimization algorithms for solving metric dimension problem'. *Graph-HOC*. Vol. 15, pp. 1-10, 2023. https://ssrn.com/abstract=4504670

VII. B. Mohamed, M. Amin. : 'Domination number and secure resolving sets in cyclic networks'. *Applied and Computational Mathematics*. Vol. 12, pp. 42-45, 2023. 10.11648/j.acm.20231202.12

VIII. B. Mohamed, M. Amin. : 'The metric dimension of subdivisions of Lilly graph, tadpole graph and special trees'. *Applied and Computational Mathematics*. Vol. 12, pp. 9-14, 2023. 10.11648/j.acm.20231201.12

IX. B. Mohamed. : 'Metric dimension of graphs and its application to robotic navigation'. *International Journal of Computer Applications*. Vol. 184, pp. 1-3, 2022. 10.5120/ijca2022922090

X. B. N. Kavitha, I. Kelkar. : 'Split and equitable domination in book graph and stacked book graph'. *International Journal of Advanced Research in Computer Science*. Vol. 8, pp. 108-112, 2017. 10.26483/ijarcs.v8i6.4475

XI. C. S. Nagabhushana, B. N. Kavitha, H. M. Chudamani. : 'Split and equitable domination of some special graph'. *International Journal of Science Technology & Engineering*. Vol. 4, pp. 50-54, 2017.

XII. F. Muhammad, L. Susilowati. : 'Algorithm and computer program to determine metric dimension of graph'. *Journal of Physics*. Vol. 1494, 012018, 2020. 10.1088/1742-6596/1494/1/012018

XIII. H. Al-Zoubi, H. Alzaareer, A. Zraiqat, T. Hamadneh, W. Al-Mashaleh. : 'On ruled surfaces of coordinate finite type'. *WSEAS Transactions on Mathematics*. Vol. 21, pp. 765–769, 2022. 10.37394/23206.2022.21.87

XIV. H. Iswadi, E. T. Baskoro, A. N. M. Salman, R. Simanjuntak. : 'The resolving graph of amalgamation of cycles'. *Utilitas Mathematica*. Vol. 83, pp. 121-132, 2010. https://api.semanticscholar.org/CorpusID:55139163

XV. I. M. Batiha, B. Mohamed. : 'Binary rat swarm optimizer algorithm for computing independent domination metric dimension problem'. *Mathematical Models in Engineering*. Vol. 10, pp. 6-13, 2024. 10.21595/mme.2024.24037

XVI. I. M. Batiha, B. Mohamed, I. H. Jebril. : 'Secure metric dimension of new classes of graphs'. *Mathematical Models in Engineering*. Vol. 10, pp. 1-6, 2024. 10.21595/mme.2024.24168

XVII. I. M. Batiha, J. Oudetallah, A. Ouannas, A. A. Al-Nana, I. H. Jebril. : 'Tuning the fractional-order PID-Controller for blood glucose level of diabetic patients'. *International Journal of Advances in Soft Computing and its Applications*. Vol. 13, pp. 1–10, 2021. https://www.i-csrs.org/Volumes/ijasca/2021.2.1.pdf

XVIII. I. M. Batiha, M. Amin, B. Mohamed, H. I. Jebril. : ‘Connected metric dimension of the class of ladder graphs’. *Mathematical Models in Engineering*. Vol. 10, pp. 65–74, 2024. 10.21595/mme.2024.23934

XIX. I. M. Batiha, N. Anakira, A. Hashim, B. Mohamed. : ‘A special graph for the connected metric dimension of graphs’. *Mathematical Models in Engineering*. Vol. 10, pp. 1-8, 2024. 10.21595/mme.2024.24176

XX. I. M. Batiha, S. A. Njadat, R. M. Batyha, A. Zraiqat, A. Dababneh, S. Momani. : ‘Design fractional-order PID controllers for single-joint robot ARM model’. *International Journal of Advances in Soft Computing and its Applications*. Vol. 14, pp. 97–114, 2022. 10.15849/IJASCA.220720.07

XXI. K. B. Murthy. : ‘The end equitable domination of dragon and some related graphs’. *Journal of Computer and Mathematical sciences*. Vol. 7, pp. 160-167, 2016.

XXII. L. Susilowati, I. Sa'adah, R. Z. Fauziyyah, A. Erfanian. : ‘The dominant metric dimension of graphs’. *Heliyon*. Vol. 6, 03633, 2020. 10.1016/j.heliyon.2020.e03633

XXIII. P. Sumathi, A. Rathi, A. Mahalakshmi. : ‘Quotient labeling of corona of ladder graphs’. *International Journal of Innovative Research in Applied Sciences and Engineering*. Vol. 1, pp. 1-12, 2017. 10.29027/IJIRASE.v1.i3.2017.80-85

XXIV. R. Alfarisi, Dafik, A. Kristiana. : ‘Resolving domination number of graphs’. *Discrete Mathematics, Algorithms and Applications*. Vol. 11, 1950071, 2019. 10.1142/S179383091950071X

XXV. R. C. Brigham, G. Chartrand, R. D. Dutton, P. Zhang. : ‘Resolving domination in graphs’. *Mathematica Bohemica*. Vol. 128, pp. 25-36, 2003. 10.21136/MB.2003.133935

XXVI. S. Kurniawati, D. A. R. Wardani, E. R. Albirri. : ‘On resolving domination number of friendship graph and its operation’. *Journal of Physics*. Vol. 1465, 012019, 2020. 10.1088/1742-6596/1465/1/012019

XXVII. R. P. Adirasari, H. Suprajitno, L. Susilowati. : ‘The dominant metric dimension of corona product graphs’. *Baghdad Science Journal*. Vol. 18, 0349, 2021. https://bsj.uobaghdad.edu.iq/index.php/BSJ/article/view/5039

XXVIII. T. Mazidah, Dafik, Slamin, I. H. Agustin, R. Nisviasari. : ‘Resolving independent domination number of some special graphs’. *Journal of Physics*. Vol. 1832, 012022, 2021. 10.1088/1742-6596/1832/1/012022